Math 100 W20 T1 Written Assignment 1

Due Wednesday September 30

Present your solutions in the clearest manner possible.

YOU MUST SHOW ALL WORK TO GET FULL MARKS.

1. Consider the following piecewise defined function,

$$g(x) = \begin{cases} x - 1 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 2 - x^2 & \text{if } 1 < x \leq 2 \\ x - 3 & \text{if } x > 2 \end{cases}$$

   (a) Evaluate the following, if they exist. If they don't exist, explain why.

      i. $\lim\limits_{x \to 1^-} g(x)$

      ii. $\lim\limits_{x \to 1} g(x)$

      iii. $\lim\limits_{x \to 2^-} g(x)$

      iv. $\lim\limits_{x \to 2^+} g(x)$

      v. $\lim\limits_{x \to 2} g(x)$

   (b) Graph $g(x)$

2. Evaluate the following limits if they exist. Justify your answers using the appropriate limit laws. If some don't exist, justify why not.

i) $\lim\limits_{t \to 1} \dfrac{t^4 - 1}{t^3 - 1}$

   Hint. $x^2 - y^2 = (x - y)(x + y)$ and $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

ii) $\lim\limits_{h \to 0} \dfrac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

iii) $\lim\limits_{x \to 5} \dfrac{5 + |x|}{5 + x}$

iv) $\lim\limits_{x \to 4} \dfrac{\sqrt{x^2 + 9} - 5}{x - 4}$

v) $\lim\limits_{x \to -\infty} \dfrac{x^6 + 1}{1 - x^4}$

3. Find all vertical and horizontal asymptote(s) of the following function if such asymptotes exist,

$$g(x) = \dfrac{\sqrt{2x^2 + 3}}{3x - 4}$$

4. Use the Squeeze Theorem to show that,

$$\lim\limits_{x \to 0} \sqrt{x^3 + x^2} \sin\left(\dfrac{\pi}{x}\right) = 0.$$